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Large angular momentum closed strings colliding with D-branes

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Abstract: We investigate colliding processes of closed strings with large angular momenta with D-branes. We give explicit CFT calculations for closed string states with an arbitrary number of bosonic excitations and no or one fermion excitation. The results reproduce the correspondence between closed string states and single trace operators in the boundary gauge theory recently suggested by Berenstein, Maldacena and Nastase.

Keywords: D-branes, Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence.

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Contents

1. Introduction

The AdS/CFT correspondence [1] relates large N quantum gauge theories and gravity or string theories in anti-de Sitter (AdS) spacetime. We can compute various quantities in Large N Yang-Mills theories by using gravity in AdS backgrounds. For example, partition functions of a four dimensional $\mathcal{N}=4$ supersymmetric Yang-Mills theory with a source term $\int J_i \mathcal{O}_i d^4x$ for gauge invariant operators $\mathcal{O}_i$ are computed as classical actions of supergravity solutions in the AdS_5 background. [2, 3] Each source J_i of operators $\mathcal{O}_i$ with conformal dimension d_i is taken into account in the supergravity computation as a boundary condition $\phi_i \sim J_i r^{4-d_i}$ on the conformal boundary at $r=0$ for a supergravity field ϕ_i corresponding to the operator $\mathcal{O}_i$. (The conformal metric $ds^2 = (dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu)/r^2$ is assumed.) This idea is based on the fact that the near horizon geometry of a large number of coinciding D3-branes is $AdS_5 \times S^5$ and gauge invariant operators in the Yang-Mills theory on the D3-branes couple to bulk fields. In the weak coupling regime, the D3-branes can be treated as branes without thickness in the flat background and the sources for gauge invariant operators $\mathcal{O}_i$ are supplied by expectation values of bulk fields $\langle\phi_i\rangle$ via brane-bulk coupling $\int \mathcal{O}_i \phi_i d^4x$. In strong coupling regime, this is re-interpreted as boundary conditions mentioned above. Although a lot of works had been done about this correspondence between boundary operators and bulk fields, only restricted types of operators had been discussed.

Recently, Berenstein, Maldacena and Nastase suggested correspondence between new class of boundary operators and bulk fields [4]. They defined a charge J associated with a U(1) subgroup of the $\mathrm{SU}(4)_R$ symmetry of four dimensional $\mathcal{N}=4$ supersymmetric Yang-Mills theory and investigated operators with large J and small $d-J$. The conformal dimension d of an operator is bounded below by the BPS bound $d \geq J$. In the context of

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bulk supergravity, J is regarded as the angular momentum along a certain direction in S^5. Therefore, near BPS operators with small $d-J$ and large J correspond to fields with near light like momentum in $AdS_5 \times S^5$. In the Penrose limit [5], which is a natural limit to study near light-like fields, the background geometry $AdS_5 \times S^5$ reduces to a PP-wave background. This background has maximal supersymmetry as $AdS_5 \times S^5$. [6, 7] Recently, string on this background was exactly solved by the Green Schwarz formalism in light-cone gauge [8]. In Ref [4], it is shown that the string spectrum on the pp-wave background is exactly reproduced as single trace operators constructed from $\mathrm{tr}(\mathcal{Z}^J)$ by insertion of fields carrying $d-J=1$.

In the weak coupling regime, in which the D3-branes are treated as branes without thickness in the flat spacetime, the suggested correspondence between string states and operators implies that closed strings with large angular momenta couple to corresponding single trace operators defined on the D3-branes. The purpose of this work is to check directly by CFT computation that such couplings between closed strings and the operators really exist. By computing disk amplitudes we explicitly show that the correspondence given in [4] is correctly reproduced as couplings among large angular momentum closed strings and open strings on D3-branes. Although explicit calculation is given only in the case that the string excitation is purely bosonic or includes one fermion oscillator, it is plausible that the correspondence is proved in the same way in the general case.

Usually, strings on the pp-wave background are studied by the light-cone GS formalism [9]. Here, to use the worldsheet CFT tecnique [10], we use the NSR formalism. In the light-cone formalism, the interpretation of the light-cone direction in the boundary CFT is not clear. This is related to a question discussed in several works: "where does the dual gauge theory live?" [11, 12, 13] To avoid this problem, and clarify the correspondence between string modes and single trace operators, we define 'spacelike light-cone' in the next section.

2. Spacelike light-cone

Let us consider coinciding D3-branes in the flat Minkowski spacetime. We denote the spacetime coordinates X^I ($I=0,\ldots,9$) where X^μ ($\mu=0,1,2,3$) are Neumann directions and X^i ($i=4,5,6,7$), $Z=(X^8+iX^9)/\sqrt{2}$ and Z^* are Dirichlet directions. We define J as the angular momentum on the Z-plane. We will discuss closed strings with large J compared with the excitation number $L_0+\widetilde{L}_0$.

In [4], the operator-string state correspondence is discussed in the framework of AdS/CFT correspondence. In this case, we should take the large 't Hooft coupling limit. The 't Hooft coupling $\eta = N g_{\mathrm{YM}}^2$ determines the radius of the AdS_5 and large η justifies the treatment of the AdS_5 background as a solution of supergravity. In this paper, we use a completely different approach. We are going to study the correspondence as interactions between closed strings and D3-branes in the flat Minkowski background. Thus, we should assume the 't Hooft coupling η (and the string coupling g_{str}) is sufficiently small. Similarly, we do not have to take the large N limit, which is necessary in the AdS/CFT framework to decouple uncontrollable effects of the quantum gravity. We emphasize that we are not

fields	A_μ	ϕ_i	$\mathcal{Z}$	$\mathcal{Z}^*$	λ_a
vertex operators	$\partial_\parallel X^\mu$	$\partial_\perp X^i$	$\partial_\perp Z^*$	$\partial_\perp Z$	$e^{-\phi/2}S^a$

Table 1: Massless fields belonging to the $\mathcal{N}=4$ vector multiplet on D3-branes and corresponding vertex operators. $\partial_\perp$ and $\partial_\parallel$ define the derivatives perpendicular or tangential to the worldsheet boundary.

going to consider AdS/CFT correspondence. We will only study the coupling between closed strings and operators in a weak coupling gauge theory.

When we compute amplitude, we consider only colliding processes satisfying the following conditions.

- Zero momentum condition: Every open string participating in processes has zero momentum.

- Least number condition: The number of open string vertices is the smallest for a given closed string state.

Because we are assuming the small string coupling constant, contribution of higher genus worldsheets are suppressed and we can take a disk amplitude as a leading contribution. If the number of open string vertices inserted on the boundary of the disk is n, the amplitude is proportional to $g_{\rm str}^{n/2}$. (The factor $g_{\rm str}$ due to one closed string vertex inserted inside the disk is included.) Therefore, the contribution of a diagram with the least number of the open string vertex dominates. Therefore we may focus on single trace operators satisfying the least number condition.

We adopt the zero momentum condition to simplify the computation of amplitudes. Of cause, if we want to consider local operators, we should represent them as superpositions of infinite number of Fourier modes. Such a treatment is necessary if we discuss correlation functions of local operators. We leave this problem for future study.

The zero momentum condition drastically reduces the open string states we have to consider. Because only massless fields can be on-shell at zero momentum, it is sufficient to consider states in the $\mathcal{N}=4$ vector multiplet. The fields and corresponding vertex operators are summarized in table 1. We have chosen boson and fermion vertices in picture 0 and in picture $-1/2$, respectively.

The zero momentum condition also constrains closed string states. By the momentum conservation, closed strings cannot carry momenta longitudinal to the D3-branes. This forbids on-shell plane waves except massless states with constant wave functions. However, we can consider closed strings carrying non-vanishing angular momenta in the following way. Vertex operators of closed strings generally have the form

$$\mathcal{O}_c = V\widetilde{V} f\left(X^I\right), \tag{2.1}$$

where V and $\widetilde{V}$ represent excitations of left and right-moving parts, respectively, which are the origin of the spin and the mass of the states, and $f(X^I)$ is an orbital wave function.

First, let us discuss the wave function $f(X^I)$. For the vertex operator to be primary operator with conformal dimension $(1,1)$, the wave function should satisfy the Laplace equation:

$$\left(\partial_i\partial_i + 2\partial_z\partial_{z^*} - M^2\right) f\left(X^i, Z, Z^*\right) = 0\,. \tag{2.2}$$

The wave function does not depend on the longitudinal coordinates X^μ because of the zero momentum condition. M in the Laplace equation (2.2) is a mass of the state determined by the conformal dimension of the oscillator parts V and $\widetilde{V}$. If we assume a plane wave function $f(X^I) = \exp(ik\cdot X)$, only zero momentum mode ($f(X^I) = \text{const}$) of massless states can satisfy (2.2). However, if we do not do so, there exist many non-trivial wave functions. What we will consider is a wave function $f_J(Z,Z^*)$ depending on Z and Z^* and satisfying the J eigenstate equation

$$(Z\partial_Z - Z^*\partial_{Z^*})f_J(Z,Z^*) = Jf_J(Z,Z^*)\,. \tag{2.3}$$

Although the function f_J grows exponentially at large $|Z|$, only the leading term in its power series expansion in Z contributes amplitudes of processes satisfying the least number condition and the divergence of f_J does not cause any problems. Therefore, we can use the wave function $f_J(Z,Z^*)$ to make the closed string vertex (2.1). We normalize the wave function such that the coefficient of the leading term is one.

$$f_J(Z,Z^*) = \sum_{k=0}^{\infty} \frac{(M^2Z^*)^k}{k!}\frac{Z^{J+k}}{(J+k)!}\,. \tag{2.4}$$

Next let us determine physical components of the oscillator factors V and $\widetilde{V}$. We discuss only massless states here. Excited states can be treated in a similar way. The vertex operator of the massless NS-NS fields in picture $(-1,-1)$ is

$$\mathcal{O}_c = V_{-1}[\zeta]\widetilde{V}_{-1}[\widetilde{\zeta}]f_J\,, \tag{2.5}$$

where the excitation parts V_{-1} and $\widetilde{V}_{-1}$ are given by

$$V_{-1}[\zeta_I] = \zeta_I V^I_{-1} = e^{-\phi}\left(\zeta_I\psi^I\right), \qquad \widetilde{V}_{-1}[\widetilde{\zeta}_I] = \widetilde{\zeta}_I\widetilde{V}^I_{-1} = e^{-\widetilde{\phi}}\left(\widetilde{\zeta}_I\widetilde{\psi}^I\right), \tag{2.6}$$

and the wave function is $f_J = Z^J$. The physical components of the polarization vectors ζ_I and $\widetilde{\zeta}_I$ are determined as follows. The BRS invariance of the vertex operator requires the following OPE to be regular.

$$j_{\rm BRS}(w)\cdot cV_{-1}f_J(z) \sim \frac{1}{w-z}\eta c\zeta^I\frac{\partial f_J}{\partial X^I}\,. \tag{2.7}$$

Because $f_J = Z^J$, This implies that $\zeta^z = 0$. The gauge transformation of this vertex operator is

$$j_{\rm BRS}(w)\cdot c\Lambda_{-1}(z) \sim \frac{1}{w-z}ce^{-\phi}\psi^I\frac{\partial\lambda}{\partial X^I}\,, \tag{2.8}$$

where λ is the gauge transformation parameter depending on the coordinates X^I and Λ_{-1} is the following operator in picture -1.

$$\Lambda_{-1} \sim e^{-2\phi}\partial\xi\lambda\left(X^I\right). \tag{2.9}$$

If we take $\lambda = Z^{J+1}$, the gauge transformation (2.8) implies that ζ^{z^*} is a gauge degree of freedom. Thus, we conclude that physical components of the vector ζ^I are ζ^μ ($\mu = 0,1,2,3$) and ζ^i ($i = 4,5,6,7$). In other word, physical degrees of freedom are $J = 0$ components of the polarization vector. The same condition is obtained for the right moving part polarization vector $\widetilde{\zeta}^I$ by using $\widetilde{j}_{\mathrm{BRS}}$. This situation is quite similar to the light-cone formalism, in which the wave function include a plane wave factor $e^{ik^-X^+}$ and physical mode oscillations are transverse to the light-cone coordinates $X^\pm$. In this sense, we refer to the Z-plane as '(spacelike) light-cone' directions.

Similar arguments can be applied to fermionic vertices. The excitation factor representing the R-vacuum in picture $-1/2$ is

$$V_{-1/2}[u_a] = u_a V^a_{-1/2} = e^{-\phi/2}(u_a S^a)\,, \tag{2.10}$$

where u_a is a 16-component polarization spinor. The OPE of the vertex and the BRS current is

$$j_{\mathrm{BRS}}(w) \cdot cV_{-1/2} f(z) \sim \frac{1}{w-z}\eta c (u\Gamma^I \overline{S}) \frac{\partial f_J}{\partial X^I}\,. \tag{2.11}$$

If we take the massless wave function $f_J = Z^J$, the BRS invariance condition demands that the spinor u satisfies

$$u\Gamma^z = 0\,. \tag{2.12}$$

This implies that eight components of u_a carrying $J = -1/2$ are physical.

3. Bosonic excitations

In the last section, we have determined the massless physical states of a closed string with 'light-cone' angular momentum. They consist of 128 bosonic and 128 fermionic massless physical states. Similarly to the ordinary light-cone GS formalism, the whole Fock space of closed string physical states is constructed by exciting these 256 vacuum states by oscillators of eight boson fields carrying $J = 0$ and eight fermions carrying $J = 1/2$. In this section, we discuss closed string states excited by only bosonic oscillators X^μ_{-n} ($\mu = 0,1,2,3$) and X^i_{-n} ($i = 4,5,6,7$). Because we can construct all the 256 ground states from one of them as states 'excited' by fermionic zero modes, we here choose one ground state as a 'vacuum' state. It will be specified in the next section. We represent the vertex operator of the 'vacuum' state of a closed string by

$$\mathcal{O}_c = \mathcal{F} f_J(Z, Z^*)\,, \tag{3.1}$$

where $\mathcal{F}$ is an operator in picture -2 consisting of matter fermions and superconfomal ghosts as will be shown in the next section.

Let us consider a non-vanishing disk amplitude including one closed string vertex (3.1) and the least number of open string vertices. Because the closed string vertex has the factor Z^J, we need at least J open string vertices $\partial_\perp Z^*$. As we see below, this amplitude does not vanish if $\mathcal{F}$ is chosen appropriately. The amplitude we would like to compute is

$$\mathcal{A}_0 = \int d^{J-1}\theta \langle c\widetilde{c}\mathcal{O}_c(\mathrm{center}) \cdot c\mathcal{O}_J(2\pi) \cdot \mathcal{O}_{J-1}(\theta_{J-1}) \cdots \mathcal{O}_2(\theta_2) \cdot \mathcal{O}_1(\theta_1)\rangle\,, \tag{3.2}$$

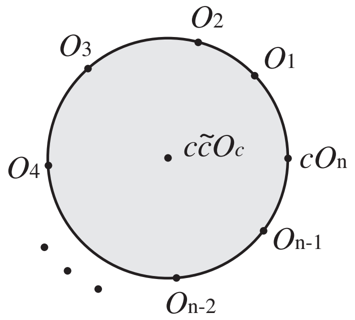

Figure 1: A worldsheet of a closed string colliding with D-branes and decaying into a large number of open strings

where $\mathcal{O}_i(\theta_i) = \partial_\perp Z^*(\theta_i)$ (figure 1). The integral $\int d^{J-1}\theta$ is defined by

$$\int d^{J-1}\theta \equiv \int_0^{2\pi} d\theta_{J-1} \int_0^{\theta_{J-1}} d\theta_{J-2} \cdots \int_0^{\theta_2} d\theta_1 \,. \tag{3.3}$$

We fix the residual $\mathrm{SL}(2,\mathbf{R})$ diffeomorphysm by putting the closed string vertex at the center of the disk and the open string vertex $\mathcal{O}_J$ at the point $\theta = 2\pi$ on the boundary. The amplitude factorizes as

$$\mathcal{A}_0 = \langle c\tilde{c}(\text{center}) \cdot c(\theta = 0)\rangle \langle \mathcal{F}\rangle \frac{J!}{J} \int d^{J-1}\theta \prod_{i=1}^{J} \langle Z(\text{center}) \cdot \partial_\perp Z^*(\theta_i)\rangle \,, \tag{3.4}$$

where $J!$ represents the number of combinations of contractions between J Zs and J $\partial_\perp Z^*$ and the $1/J$ factor reflects the ambiguity of the choice of the position-fixed vertex $\mathcal{O}_J$. Because the conformal ghost factor $\langle c\tilde{c}\cdot c\rangle$ and the matter fermion and superconformal ghost factor $\langle \mathcal{F}\rangle$ give just constant factors, let us focus on the contribution from contractions of Z and Z^*. The Green function of a scalar field on the unit disk with Dirichlet boundary condition is

$$\langle \phi(w)\phi(z)\rangle = -\log|w - z| + \log|1 - wz^*|. \tag{3.5}$$

From this Green function, we easily obtain

$$\left\langle Z(0)\partial_\perp Z^*(e^{i\theta})\right\rangle = 1 \,. \tag{3.6}$$

By substituting (3.6) into (3.4) and carrying out the integrations, we obtain

$$\mathcal{A}_0 = \langle c\tilde{c} \cdot c\rangle \langle \mathcal{F}\rangle (J-1)! I_{J-1}(2\pi). \tag{3.7}$$

The function $I_n(x)$ is defined as an integration over n ordered variables in the interval $[0,x]$.

$$I_n(x) = \int_0^x dx_n \int_0^{x_n} dx_{n-1} \cdots \int_0^{x_2} dx_1 \cdot 1 = \frac{x^n}{n!} \,. \tag{3.8}$$

The amplitude (3.7) is interpreted as a coupling between the closed string state (3.1) and the single trace operator $\mathrm{tr}(\mathcal{Z}^J)$.

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Let us consider excitation modes of a closed string. We first discuss excitation by oscillators of Dirichlet directions X^i_{-n} and $\widetilde{X}^i_{-n}$ ($i = 4, 5, 6, 7$). The excitation by oscillators of Neumann directions X^μ_{-n} and $\widetilde{X}^\mu_{-m}$ ($\mu = 0, 1, 2, 3$) will be mentioned later. An excitation by X^i_{-n} ($\widetilde{X}^i_{-n}$) is realized by adding a factor $\partial^n X^i$ ($\widetilde{\partial}^n X^i$) to the massless vertex operator. Here we discuss only the case with one left-moving excitation X^i_{-m} and one right-moving excitation $\widetilde{X}^j_{-n}$. Generalization to the case with arbitrary number of excitations is straightforward. The vertex operator is

$$\mathcal{O}_c = \frac{1}{m!}\partial^m X^i \cdot \frac{1}{n!}\widetilde{\partial}^n X^j \cdot \mathcal{F} \cdot Z^J . \tag{3.9}$$

In general, vertex operators obtained in this way are not BRS invariant and we need to add extra terms in order to make them BRS invariant. For example, the massive wave function f_J is not simply Z^J but includes infinite terms of the form $Z^{J+k}Z^{*k}$ ($k = 1, 2, \ldots$). However, these terms do not contribute amplitudes we will consider below because these extra terms consist of larger number of fields than the leading term Z^J and some of them do not have partner operators to be contracted with in amplitudes satisfying the least number condition. We also need to modify the excitation part $V\widetilde{V} \sim \partial^m X^i \widetilde{\partial}^n X^j \mathcal{F}$ because operators of the form $\partial^n X^I$, $\widetilde{\partial}^n X^I$ and their products are in general not primary operators. We have to add extra terms to the vertex operators. In the computation of amplitudes given below, we neglect such extra terms. Although we can explicitly show for some simple examples that they indeed do not contribute the amplitude, in order to prove the irrelevance of these terms for arbitrary cases, we need detailed investigation of structure of vertex operators of circular wave closed strings. Unfortunately, we have not obtained a strict proof at this point and we just suppose that these terms do not affect our analysis of the operator-string state correspondence.

If we replace $\mathcal{O}_c$ in (3.4) by the vertex (3.9), the amplitude vanishes. To obtain non-vanishing amplitude, we need two vertices $\partial_\perp X^i(\varphi)$ and $\partial_\perp X^j(\varphi')$ on the boundary in addition to the J vertices $\partial_\perp Z^*(\theta_i)$. We insert these two extra vertices into the p-th and q-th intervals (figure 2). Namely, the angles φ and φ' satisfy

$$\theta_{p+1} > \varphi > \theta_p , \qquad \theta_{q+1} > \varphi' > \theta_q . \tag{3.10}$$

Contribution from the contractions between these extra open string vertices and excitation factors in the closed string vertex (3.9) is obtained from the correlation function (3.5).

$$\frac{1}{m!}\left\langle \partial^m X^i(0)\partial_\perp X^{i'}(e^{i\varphi})\right\rangle = \delta^{ii'}e^{-im\varphi} , \qquad \frac{1}{n!}\left\langle \widetilde{\partial}^n X^j(0)\partial_\perp X^{j'}(e^{i\varphi'})\right\rangle = \delta^{jj'}e^{in\varphi'} . \tag{3.11}$$

The amplitude is given by

$$\mathcal{A} = \langle c\widetilde{c} \cdot c\rangle\langle\mathcal{F}\rangle\frac{J!}{J}\int d^{J-1}\theta \int_{\theta_p}^{\theta_{p+1}} d\varphi \int_{\theta_q}^{\theta_{q+1}} d\varphi' e^{-im\varphi}e^{in\varphi'} . \tag{3.12}$$

(If the directions of the polarizations of the two excitations coincide, there are one more term coming from the other contraction of boson fields. We here assume $i \neq j$.) When the number J is very large and both $|p-q|$ and $J - |p-q|$ are the same order of magnitude

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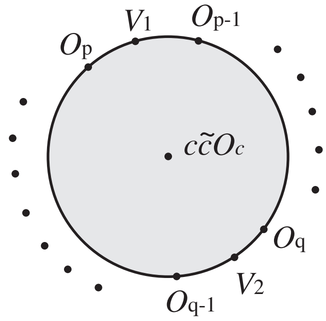

Figure 2: A worldsheet with two extra insertions of vertices $V_1 = \partial_\perp X^i$ and $V_2 = \partial_\perp X^j$.

as J, this integral can be estimated with the help of the dilute gas approximation. [4] (see appendix A) We replace the integration over $J-1$ angular variables θ_k by a constant factor $I_{J-1}(2\pi)$ and fixed even-interval points $\theta_k = 2\pi k/J$ and we have

$$\mathcal{A} = \frac{(2\pi)^2}{J^2} e^{-im2\pi p/J} e^{in2\pi q/J} \mathcal{A}_0 \,. \tag{3.13}$$

This amplitude implies that the existence of a coupling between the closed string state (3.9) and the single trace operator

$$\sum_{p,q=1}^{J} e^{-2\pi i(mp/J - nq/J)} \operatorname{tr}\left(\mathcal{Z}^{J-p}\phi_i \mathcal{Z}^{p-q}\phi_j \mathcal{Z}^q\right) . \tag{3.14}$$

Of cause, when $m \neq n$, this amplitude vanishes because of the summation of p and q. Generalization to the case with arbitrary number of excitations are straightforward. This result coincide with the suggestion in [4].

In the argument above, we have assumed $m, n \geq 1$. It is also possible to consider 'level 0' excitation by adding a factor X^i to the vertex operator. The contraction between this factor and a vertex on the boundary is

$$\left\langle X^i(0)\partial_\perp X^i\left(e^{i\theta}\right)\right\rangle = 1 \,. \tag{3.15}$$

Thus, this corresponds to an insertion of the scalar field ϕ_i in the single trace operator without phase factor like

$$\sum_{p=1}^{J} \operatorname{tr}\left(\mathcal{Z}^{J-p}\phi_i \mathcal{Z}^p\right) . \tag{3.16}$$

The factor X^i in the closed string vertex changes the wave function f_J rather than the oscillator parts $V\widetilde{V}$. This is consistent with the fact that the operator (3.16) is a chiral primary and is related to a Kaluza-Klein mode of supergravity fields on $\mathbf{S}^5$ via the AdS/CFT correspondence.

Inclusion of excitations by oscillators of Neumann directions is straightforward. The Green function of a scalar field on the unit disk with Neumann boundary condition is

$$\langle\phi(w)\phi(z)\rangle = -\log|w-z| - \log|1-wz^*| + \frac{1}{2}\left(|w|^2 + |z|^2\right) . \tag{3.17}$$

From this, we can obtain correlation functions between operator $\partial^n X^\mu$ or $\tilde{\partial}^n X^\mu$ at the center of the disk and vertex operator $\partial_{||} X^\mu$ on the boundary as

$$\frac{1}{n!}\left\langle \partial^n X^\mu(0)\partial_{||}X^\nu(e^{i\theta})\right\rangle = -i\eta^{\mu\nu}e^{-in\theta}\,, \qquad \frac{1}{n!}\left\langle \tilde{\partial}^n X^\mu(0)\partial_{||}X^\nu(e^{i\theta})\right\rangle = i\eta^{\mu\nu}e^{in\theta}\,. \quad (3.18)$$

Therefore, an excitation $\partial^n X^\mu$ corresponds to an insertion of the gauge field A_μ with a phase factor like

$$\sum_{p=1}^{J} \mathrm{tr}(\cdots A_\mu \cdots) e^{-2\pi inp/J}\,, \qquad (3.19)$$

where p is a position of A_μ inserted. A right moving excitation $\tilde{\partial}^n X^\mu$ corresponds to a similar insertion with an opposite phase. The operator (3.19) is gauge invariant if $n \geq 1$. Indeed we can rewrite (3.19) up to constant factor as

$$\sum_{p=1}^{J} \mathrm{tr}\left(\cdots [A_\mu, \cdot] \cdots\right) e^{-2\pi inp/J}\,. \qquad (3.20)$$

Because we assume vanishing open string momenta, the commutator $[A_\mu, \mathcal{O}]$ is equivalent to the covariant derivative $D_\mu \mathcal{O} = \partial_\mu \mathcal{O} + [A_\mu, \mathcal{O}]$ and is gauge covariant. Therefore, the trace (3.20) is a gauge invariant operator.

If $n = 0$, however, the expression (3.20) identically vanishes. We can also see this on the CFT side as follows: Let us add a level zero excitation factor X^μ to a closed string vertex and compute the amplitude. Unlike the case of Dirichlet boundary condition, it has vanishing contraction with a vertex on the boundary.

$$\left\langle X^\mu(0)\partial_{||}X^\nu\left(e^{i\theta}\right)\right\rangle = 0\,. \qquad (3.21)$$

Therefore, we obtain a vanishing coupling between the closed string and the operator with gauge field insertion without phase factor.

4. Fermionic excitations

Let us discuss fermion excitations of closed strings. In the previous section, we have not specified the vacuum state (3.1). Let us begin with determining it. Among 256 ground states, only one state gives non-vanishing one point function $\langle \mathcal{F} \rangle$. To determine such a $\mathcal{F}$, we can use symmetries of the system. The rotational symmetry transverse to the spacelike light-cone is $\mathrm{SO}(1,3) \times \mathrm{SO}(4)$. By taking T-duality along X^4, X^5, X^6 and X^7, this symmetry is enhanced into $\mathrm{SO}(1,7)$. Let us refer to this symmetry as $\widetilde{\mathrm{SO}(1,7)}$. In this section, we use this T-dual picture to make the $\widetilde{\mathrm{SO}(1,7)}$ symmetry manifest. To have non-vanishing $\langle \mathcal{F} \rangle$, we should take an $\widetilde{\mathrm{SO}(1,7)}$ singlet as $\mathcal{F}$. There are two singlet physical states. One is an NS-NS state ($\mathcal{F} = e^{-\phi-\tilde{\phi}} \sum_{\mu=0}^{7} \psi_\mu \tilde{\psi}^\mu$) and the other is an R-R state ($\mathcal{F} = e^{-\phi/2-3\tilde{\phi}/2} SC\Gamma^z \tilde{S}$). However, one point function of the R-R singlet state vanishes

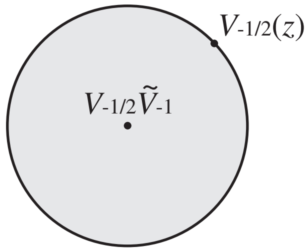

Figure 3: A worldsheet with one fermion vertex insertion. All the insertions of bosonic operators on the boundary are omitted.

because it carries angular momentum $J = 1$. Therefore, the 'vacuum state' should be the NS-NS singlet state. In the original picture before taking the T-duality, this operator is represented as

$$\mathcal{F} = e^{-\phi-\widetilde{\phi}} \left(\sum_{\mu=0}^{3} \psi_\mu \widetilde{\psi}^\mu - \sum_{i=4}^{7} \psi^i \widetilde{\psi}^i \right) . \tag{4.1}$$

This state should be identified with the unique ground state of a closed string in the PP-wave background [8]. In the GS formalism, all the other massless states are generated from it by acting the zero modes of the GS fermions ψ^a in $\mathbf{8_s}$ representation of SO(1,7). They belong to $(\mathbf{8_v} + \mathbf{8_c}) \times (\mathbf{8_v} + \mathbf{8_c})$ of SO(1,7) and are decomposed into irreducible representations as follows.

$$\begin{array}{ll} \text{NS-NS} \;\; \mathbf{1} + \mathbf{28} + \mathbf{35_v}\,, & \quad \text{NS-R} \;\; \mathbf{8_s} + \mathbf{56_s}\,, \\ \text{R-R} \;\; \mathbf{1} + \mathbf{28} + \mathbf{35_c}\,, & \quad \text{R-NS} \;\; \mathbf{8_s} + \mathbf{56_s}\,. \end{array} \tag{4.2}$$

This completely coincides with a set of the antisymmetric products of from zero to eight $\mathbf{8_s}$'s. Therefore, we guess the corresponding operators in $\mathcal{N} = 4$ SYM are ones with gaugino insertions without phase factor. As the simplest case, let us consider one gaugino insertion. The relevant closed string state must belongs to the $\mathbf{8_s}$ representation of SO(1,7). The vertex operator of the $\mathbf{8_s}$ state in the R-NS sector is

$$\mathcal{O}_c = e^{-\phi/2-\widetilde{\phi}} \overline{\zeta}_{\dot{a}} (\Gamma_\mu)^{\dot{a}}{}_b V^b_{-1/2} \widetilde{V}^\mu_{-1}\,, \tag{4.3}$$

where the physical state condition for the polarization spinor $\overline{\zeta}_{\dot{a}}$ is $\overline{\zeta}\Gamma^z = 0$. The amplitude we would like to compute is

$$\left\langle \mathcal{O}_c \partial_\perp Z^* \cdots \partial_\perp Z^* V_{-1/2}[\lambda_a] \partial_\perp Z^* \cdots \partial_\perp Z^* \right\rangle , \tag{4.4}$$

where the gaugino vertex $V_{-1/2}[\lambda_a]$ is inserted into the p-th interval $[\theta_p, \theta_{p+1}]$. Because the gaugino has no momentum, any conditions are not imposed on λ_a.

The amplitude is decomposed into several sectors. The sectors of boson fields X^I and the conformal ghost are the same with what is discussed in the last section (see figure 3). In addition to it, we now have the superconformal ghost sector

$$\left\langle \, e^{-\phi(0)/2} e^{-\widetilde{\phi}(0)} e^{-\phi(z)/2} \right\rangle \propto \frac{1}{z^{1/4}}\,, \tag{4.5}$$

and the matter fermion sector

$$\left\langle \overline{\zeta}_{\dot{a}}(\Gamma^i)^{\dot{a}}{}_b S^b(0)\widetilde{\psi}^i(0)\lambda_a S^a(z)\right\rangle \propto \frac{1}{z^{3/4}}\left(\overline{\zeta}C\lambda\right), \tag{4.6}$$

where C is the charge conjugation matrix in ten-dimension. Because the closed string polarization spinor $\overline{\zeta}$ carries $J = -1/2$, only half of λ with $J = 1/2$ participate in this process. These factors are combined into

$$\mathcal{A} \propto \int d^{J-1}\theta \int_{[p]} dz \frac{1}{z}\left(\overline{\zeta}C\lambda\right), \tag{4.7}$$

where $[p]$ is the arc between two points θ_{p+1} and θ_p on the boundary. By means of the dilute gas approximation, we obtain the following p-independent result.

$$\mathcal{A} \propto \overline{\zeta}C\lambda\,. \tag{4.8}$$

This implies that the closed string state (4.3) couples to the gauge invariant operator

$$\sum_{p=1}^{J} \mathrm{tr}\left(\mathcal{Z}^{J-p}\lambda^{J=1/2}\mathcal{Z}^p\right), \tag{4.9}$$

where $\lambda^{J=1/2}$ represent eight components satisfying $\lambda\Gamma^{z^*} = 0$. Because the NS-R closed string vertex

$$\mathcal{O}_c = e^{-\phi/2-\widetilde{\phi}}\overline{\zeta}_{\dot{a}}(\Gamma_\mu)^{\dot{a}}{}_b V^\mu_{-1}\widetilde{V}^b_{-1/2}\,, \tag{4.10}$$

also couples to the operator (4.9), a closed string state coupled to the boundary operator (4.9) is a linear combination of the R-NS state (4.3) and the NS-R state (4.10). The other combination of these two states would couples to a boundary operator with seven fermion insertions, which also belongs to the $\mathbf{8_s}$ representation of $\widetilde{\mathrm{SO}(1,7)}$. In this way, we can establish a relation between the 256 physical massless states of a closed string and operators with gaugino insertions without phase factors.

Next, let us discuss excitations by fermion non-zero modes. In NSR formalism, the excitation is realized by a contour integral of a fermion vertex. The level n fermion excitation of a vertex $\mathcal{O}_c$ is given by

$$\mathcal{O}'_c(0) = \oint dw \frac{1}{w^n} V_{1/2}[u^a](w)\mathcal{O}_c(0)\,. \tag{4.11}$$

We here neglect the extra terms necessary for the vertex to be BRS invariant as we have done for bosonic excitations. The vertex operator $V_{1/2}[u_a]$ in picture $1/2$ is defined by

$$V_{1/2}[u_a] = e^{-\phi/2}u_a(\Gamma^z)^a{}_{\dot{b}}\overline{S}^{\dot{b}}\partial Z^* + \cdots\,, \tag{4.12}$$

where '$\cdots$' means terms irrelevant to our calculation. When $\mathcal{O}_c$ is the ground state vertex (3.1) and $n = 0$, $\mathcal{O}'_c$ is fermion vertex given in (4.3). As a simplest case, let us consider one fermion insertion given by (4.11) with $\mathcal{O}_c$ in (3.1). In what follows, we omit all the

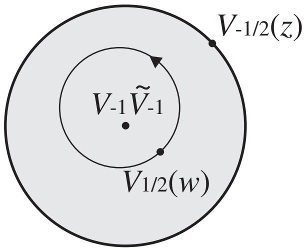

Figure 4: A world sheet with one fermion insertion on the boundary and closed string vertex with a fermionic excitation. All the insertions of bosonic operators on the boundary are omitted.

bosonic open string operators because they just give the factor we have discussed in the previous section. The relevant part of the amplitude is

$$\mathcal{A} \propto \int d^{J-1}\theta \int_{[p]} dz \oint dw \frac{1}{w^n} \eta_{\mu\nu} \left\langle V^{\mu}_{-1}(0) V^{\nu}_{-1}(\infty) V_{+1/2}[u_a](w) V_{-1/2}[\lambda_a](z) \right\rangle \tag{4.13}$$

with the polarization u satisfying

$$u\Gamma^{z^*} = 0\,. \tag{4.14}$$

Although we also need boson excitations to satisfy the level matching condition, we omit them because they just give the factor we obtained in the last section (see figure 4).

The matter fermion sector is

$$\left\langle \psi_\mu(0)\widetilde{\psi}^\mu(0) u\Gamma^z \overline{S}(w)\lambda S(z) \right\rangle \propto \frac{w+z}{w^{1/2} z^{1/2} (w-z)^{5/4}} (u\Gamma^z C\lambda). \tag{4.15}$$

Because of the coupling $u\Gamma^z C\lambda$, only $J = 1/2$ components of λ satisfying $\lambda\Gamma^{z^*} = 0$ participate in the interaction. The superconformal ghost sector is

$$\langle e^{-\phi(0)} e^{-\widetilde{\phi}(0)} e^{\phi(w)/2} e^{-\phi(z)/2} \rangle \propto \frac{w^{1/2}(w-z)^{1/4}}{z^{1/2}}\,. \tag{4.16}$$

Furthermore, we have a factor $1/w$ coming from a contraction between ∂Z^* in $V_{1/2}$ and $f_J(Z)$. The correlation function is obtained as a product of these factors and boson factor discussed in the previous section. By using the dilute gas approximation, the amplitude is obtained as

$$\mathcal{A} \propto \int d^{J-1} d\theta \int_{[p]} dz \oint dw \frac{1}{w^n} \frac{z+w}{wz(z-w)} (u\Gamma^z C\lambda) \propto e^{-2\pi i n p/J} (u\Gamma^z C\lambda)\,. \tag{4.17}$$

Now we are omitting the factor from boson excitations. The amplitude has a phase factor $e^{-2\pi i n p/J}$ depending on the position of the fermion insertion. This implies that the closed string vertex $\mathcal{O}'_c$ couples to the boundary operator

$$\sum_{p=1}^{J} \mathrm{tr}\left(\cdots \lambda^{J=1/2} \cdots \right) e^{-2\pi i n p/J}\,, \tag{4.18}$$

where p is the position of the $\lambda^{J=1/2}$ insertion. This is exactly what suggested in [4].

5. Conclusions

In this paper we investigated large angular momentum closed strings colliding with D3-branes and decaying into large number of open strings. As couplings between closed strings and gauge invariant operators on D3-branes, the string states-boundary operators correspondence suggested in [4] is reproduced completely for states without fermionic excitation. Concerning states including fermionic excitations, we gave explicit computation of amplitudes only for the case of one fermion excitation. It is plausible that the correspondence for general states would be reproduced in the same way.

We briefly comment on fields carrying $d-J=2$. In processes consistent with the least number condition, only $d-J=0$ ($\mathcal{Z}$) and $d-J=1$ fields (A_μ, ϕ_i, $\lambda^{J=1/2}$) participate in the interactions. Although single trace operators including $d-J=2$ fields ($\mathcal{Z}^*$, $\lambda^{J=-1/2}$) also have non-vanishing couplings with closed strings, the operators 'decay' into other operators consisting of less number of fields. [4] For example, let us consider the operator $\mathrm{tr}(\mathcal{Z}^*\mathcal{Z}^{J+1})$. It is expressed as insertion of $J+1$ $\partial_\perp Z^*$ and one $\partial_\perp Z$ on the disk boundary. Because the vertex $\partial_\perp Z$ can be contracted with one of $\partial_\perp Z^*$, the operator cannot satisfy the least number condition.

After ref. [4], many works has appeared discussing the pp-wave limit of orbifold [14]–[21] and open strings in the pp-wave background [22, 23, 24] along the line of [4]. We hope that our CFT analysis is also useful to study these cases.

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A. Dilute gas approximation

Consider the following integral:

$$A=\int_0^x d^n x\int_{x_k}^{x_{k+1}}dyf(y)\,, \tag{A.1}$$

where $\int_0^x d^n x$ is defined by

$$\int_0^x d^n x=\int_0^x dx_n\int_0^{x_n}dx_{n-1}\cdots\int_0^{x_2}dx_1\,. \tag{A.2}$$

The function $f(y)$ depends only one variable y. Integration over variables x_i gives

$$A=\int_0^x I_k(y)I_{n-k}(x-y)f(y)dy=\int_0^x\rho(y)f(y)dy\,. \tag{A.3}$$

where the density function $\rho(y)$ is defined by

$$\rho(y)=I_k(y)I_{n-k}(x-y)=\frac{y^k(x-t)^{n-k}}{k!(n-k)!}\,. \tag{A.4}$$

This function is expanded around its maximum point $y = y_0$ as

$$\rho(y_0 + \epsilon) = \rho(y_0)\left(1 - \frac{\epsilon^2}{2x^2}\frac{n^3}{k(n-k)} + \mathcal{O}\left(\epsilon^3\right)\right), \qquad y_0 = \frac{k}{n}x\,. \tag{A.5}$$

If n is sufficiently large, we can estimate the integral (A.3) by steepest dissent approximation and we obtain

$$A = \frac{x}{n}\sqrt{\frac{2\pi k(n-k)}{n}}\rho(y_0)f(y_0)\,. \tag{A.6}$$

On the other hand, if both k and $n-k$ are the same order of magnitude as n, the maximum of the function $\rho(y)$ is estimated by the Stirling formula $z!/z^z = \sqrt{2\pi z}e^z(1+\mathcal{O}(z^{-1}))$.

$$\rho(y_0) = \frac{k^k}{k!}\frac{(n-k)^{n-k}}{(n-k)!}\frac{n!}{n^n}\frac{x^n}{n!} = \sqrt{\frac{n}{2\pi k(n-k)}}\frac{x^n}{n!}\left(1+\mathcal{O}\left(n^{-1}\right)\right). \tag{A.7}$$

By substituting (A.7) into (A.6), we obtain a simple expression for A.

$$A = \frac{x}{n}f\left(\frac{kx}{n}\right)I_n(x)\,. \tag{A.8}$$

With the help of this formula, we can easily estimate the following integral of a function depending on two variables.

$$B = \int_0^x d^n x \int_{x_k}^{x_{k+1}} dy \int_{x_l}^{x_{l+1}} dz f(y,z)\,. \tag{A.9}$$

We can carry out x_i integration to obtain

$$B = \int_0^x dy \int_0^y dz I_{n-k}(x-y)I_{k-l}(y-z)I_l(z)f(y,z)\,. \tag{A.10}$$

By the formula (A.8) we can carry out z integration and the result is

$$B = I_{n-k}(x-y)I_k(y) \times \frac{y}{k}f\left(y, \frac{ly}{k}\right). \tag{A.11}$$

We again use the formula (A.8) to integrate over y and obtain

$$B = \frac{x^2}{n^2}f\left(\frac{kx}{n}, \frac{lx}{n}\right)I_n(x)\,. \tag{A.12}$$

Generalization to the case of the integrated function depending on an arbitrary number of variables is

$$\int_0^x d^n x \prod_{i=1}^{p} \int_{x_{k_i}}^{x_{k_i+1}} dy_i f(y_1, y_2, \ldots, y_p) = \frac{x^p}{n^p} f\left(\frac{k_1 x}{n}, \frac{k_2 x}{n}, \ldots, \frac{k_p x}{n}\right) I_n(x) \tag{A.13}$$

This formula is expressed by the statement: "we can replace the x_i integrals with constant factor $I_n(x)$ and fixed even-interval points $x_k = (k/n)x$."

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